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Perturbations of a Transonic Flow with Vanishing Shock Waves

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Introduction

THE computation of transonic flows using potential theory is fairly commonplace, although the computation cost is still high and, consequently, considerable effort has been expended in trying to reduce the total costs of the necessary calculations. One approach is to improve the numerical algorithms so that the central processor time for a given computation is reduced. A second approach is to attempt to extend the usefulness of a relatively few expensive high-grade numerical computations by extracting as much information as possible from the computed results. This second approach is the driving force behind the development of the transonic perturbation theory.[1,2] A consequence of the theory is that nonlinear transonic solutions can be interpolated or extrapolated to give a range of solutions if only two solutions are initially known.

The main problem in developing a perturbation theory for transonic flow with a shock wave is that the basic premise of small perturbations breaks down if the shock wave changes location. In Ref. 1 a strained coordinate technique is developed in which the shock location is kept invariant in the new coordinate system. A restriction of the theory is that shock waves cannot be generated or destroyed during the perturbation. The perturbation equations are linear in the strained coordinates but on the return to physical coordinates the correct nonlinearity of the problem is apparent. One advantage of the linearity of the perturbation equation is that superposition of the effects of perturbations in several different parameters is possible, a feature which is very useful in design applications. Although the utility of the basic perturbation theory and its unsteady development, the indicial method, has been proven, the restriction that shock waves cannot be generated or destroyed during the perturbation invalidates the application of the theory to several important areas in both steady and unsteady transonic aerodynamics. This Note concerns the results of a preliminary investigation[3] into the behavior of the perturbation theory in areas where the shock wave will vanish during a small perturbation.

Analysis

The basic equation used in the present investigation is the transonic small disturbance equation, which is written in the form[4]

$$\phi_{xx} + \phi_{yy} = \phi_x \phi_{xx} \tag{1}$$

where ϕ is a scaled perturbation velocity potential related to the perturbation velocity potential. In this version of the transonic small disturbance equations, sonic conditions exist when

$$u = \phi_x = 1 \tag{2}$$

The usual boundary conditions for Eq. (1) are applied. Equation (1) and its boundary conditions can be written in integral form[3] to give

$$u - (u^2/2)\cos^2\sigma_s = I_L + I_s = F \tag{3}$$

where I_L is a known function of the tangency boundary conditions and I_s a surface integral. It is not necessary to know the precise form of I_L and I_s except to note that both are continuous functions of (x,y) and have continuous first derivatives; thus, the detailed definition is not given here. The angle the shock makes with the y axis is σ_s.

Equation (3) must be solved subject to the following regularity conditions which eliminate expansion shocks.

$$\left(\frac{\partial F}{\partial x}\right)_{x_0} = 0 \tag{4a}$$

$$(F)_{x_0} = 1/2m^2 \tag{4b}$$

Here, $x = x_0$ is the switching point for a zero strength jump and $m = \cos\sigma_s$. If Eqs. (4a) and (4b) are also satisfied at a decelerating point in the flow, then there is no compression shock and the flow is shock-free. A detailed description of the integral equation method is given in Ref. 4.

The rationale and detailed description of the transonic perturbation method are given by Nixon.[1,2] The basic integral equations for the perturbation problem are as follows.

$$u_1(1-u_0) = I_{L_1} + I_{s_1}(u_1,u_0) + \delta x_s I_f(u_0) \tag{5}$$

where, if x' is a strained coordinate, given by

$$x = x' + \delta x_s x_1(x')$$

where $x_1(x')$ is a specified straining function, then

$$u_1(x',y) = \phi_{1x'}(x',y)$$

$$u_0(x',y) = \phi_{0x'}(x',y)$$

I_{L1} is a known continuous function of the perturbation boundary conditions and I_f is a known function of u_0. The quantities I_{s1} and I_f are not required in detail in the present work and the reader is referred to Ref. 1 for further information. The subscript *1* denotes a perturbation quantity.

Equation (5) must be solved subject to the following regularity condition.

$$[I_{L_1} + I_{s_1} + \delta x_s I_f(u_0)]_{x=x_0} = 0 \tag{6}$$

where $x = x_0$ denotes the location of the switching point. Equation (6) is sufficient to give δx_s. The main restriction on the perturbation theory is that shock waves cannot be generated or destroyed during a perturbation.

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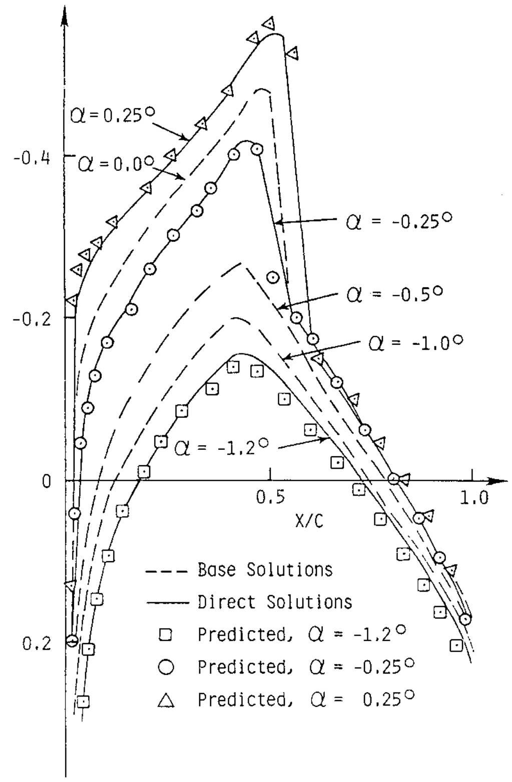

Fig. 1 Interpolation theory for the upper surface of an NACA 64A006 airfoil at $M_\infty = 0.85$.

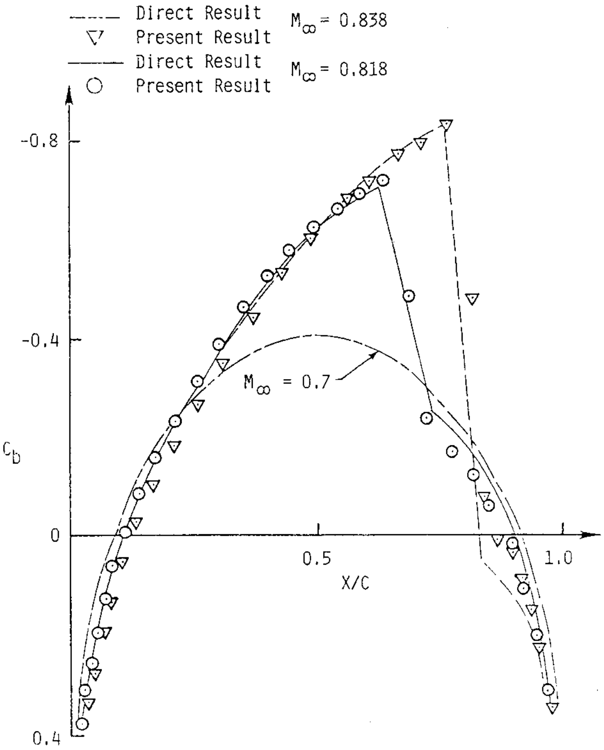

Fig. 2 Pressure distribution around a 10% biconvex airfoil.

It can be implied (see Ref. 3) that there is a smooth transition from a weak compression shock through critical to a weak expansion shock, and from a shocked flow to a critical flow and from a critical flow to a subcritical flow. As noted earlier, the potential equation can admit expansion shocks, but these must be removed by the imposition of an entropy condition at the creation of the expansion shock; at a critical flow the entropy condition is also necessary and, hence, the equation set is discontinuous at a point when a compression shock is reduced to a critical condition because an additional entropy condition is required at the location of the compression shock.

Hence, a unified theory valid from subcritical through supercritical flow is not possible; however, a piecewise perturbation theory, one piece for supercritical flows and one for subcritical flows, can be constructed. Furthermore, since the critical flow is a smooth boundary, the interpolation theory described in Ref. 2 can be constructed using three solutions, namely, one subcritical solution and one supercritical solution with the critical solution as the second solution for both regions. The problem can be simplified further since the critical solution can be estimated by extrapolation from two subcritical solutions or two supercritical solutions.

Results

It has been stated above that a piecewise application of the perturbation theory can be constructed and in this section applications of the interpolation theory are given. In these examples three solutions are assumed known and in the present case there are two subcritical solutions and one supercritical solution. This is regarded as a more useful test case since a subcritical solution is generally easier to obtain than a supercritical solution.

The critical solution is obtained by extrapolating the subcritical solutions until the peak velocity on the airfoil is sonic. The computer code used in these calculations is the steady flow part of the Ballhaus-Goorjian[5] computer code LTRAN2.

The first example is for the upper surface of a NACA 64A006 at $M_\infty = 0.85$. The subcritical solutions are at $\alpha = -1.0$ deg and $\alpha = -0.5$ deg, while the supercritical solution is at $\alpha = 0.0$ deg. The results of the piecewise perturbation method are compared to direct solutions in Fig. 1 and it may be seen that the agreement is good.

Before the advent of large computers several attempts were made to derive "compressibility corrections" that relate a compressible flow to an incompressible flow. The most well-known of these techniques is the Prandtl-Glauert factor. A more recent factor is the first approximation of Nixon and Hancock[4] which relates the perturbation velocity u to the velocity computed by the Prandtl-Glauert formula, U_{PG} by the formula

$$u - \frac{(\gamma+1)M^2}{(1-M_\infty^2)}\frac{u^2}{4} = U_{PG} \tag{7}$$

This formula is accurate for flows up to critical and it is obvious that, if U_{PG} is known, then the necessary two subcritical solutions can be constructed using Eq. (7). Since U_{PG} is related to the incompressible solution, it then follows that the interpolation method will give a complete range of solutions as the Mach number increases if the incompressible solution and one supercritical solution are known.

An example of this technique is illustrated in Fig. 2, where the flow around a 10% biconvex airfoil at various Mach numbers is computed using the incompressible result and a supercritical result at $M_\infty = 0.828$.

Conclusions

This Note is concerned with an investigation into the behavior of the transonic perturbation theory when shock waves vanish. It is shown that, when shock waves vanish, a piecewise application of the perturbation theory is necessary at that point since the equation set is discontinuous there. It is also shown that the range of transonic solutions can be constructed if the incompressible solutions and one transonic solution are known.

Acknowledgment

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Upstream Influence in Conically Symmetric Flow

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Introduction

In sharp fin-induced flows, where a swept planar shock wave interacts with a turbulent boundary layer, experimental results[1-3] show that outside of an inception zone near the fin leading edge, the interaction footprint is conically symmetric. Conically symmetric means that surface features, such as the line of upstream influence, lie along rays which intersect the trace of the inviscid shock wave at a common origin. This, and the coordinate system used in this Note, are shown in Fig. 1. The origin, as shown, may be a virtual one offset by distance ΔL_s from the leading edge or the leading edge itself. In the conically symmetric regime, the spanwise growth of upstream influence, L_{u_n}, can be expressed in normalized form as

$$L_{u_n}/(L_s+\Delta L_s)=\tan(\beta_u-\beta_s) \qquad (1)$$

where β_u and β_s are the angles of the line of upstream influence and the shock wave, respectively (Fig. 1).

Although this is a very simple formulation, it cannot be used in a predictive way unless β_u is known (β_s is calculated from the freestream Mach number M_∞ and the fin angle of attack α). The focus of the present work is to examine the available experimental data and determine the relationship between β_u and β_s. The data used are from the four studies given in Table 1.

The Mach 2 and 3 tests were made under adiabatic wall temperature conditions. Those at Mach 6 were made with a cooled wall (the wall to recovery temperature ratio was 0.5). The data of McCabe[6] ($M_\infty=2.95$), Peake[7] ($M_\infty=2$), Kubota[8] ($M_\infty=2.36$, 2.41), and Lowrie[9] ($M_\infty=3.44$) were also examined, but were judged to be within the inception zone, and thus have not been used.

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Discussion of Results

Upstream influence was determined from wall-pressure data at $M_\infty=2$ and 6 and from pressure data and surface streak patterns at $M_\infty=3$. In all cases, inviscid shock theory was used to calculate β_s. Figure 2 shows β_u vs β_s. It is estimated that the accuracy of β_u is ± 1 deg. The larger errors are for small α and are due more to uncertainty in locating the upstream influence line than in measuring a smaller angle. The flagged data at Mach 3 were obtained recently[10] in tests carried out under the same freestream conditions as Ref. 5. The hatched lines are discussed shortly.

At Mach 6, straight lines can be fitted through the upstream influence points and the fin leading edge at all α, indicating that the entire flowfield is conically symmetric. This can also be seen in the sketches of the surface streak patterns, shown in Ref. 1. With $\Delta L_s=0$ Eq. (1) becomes

$$L_{u_n}/L_s=\tan(\beta_u-\beta_s) \qquad (2)$$

With results from only one set of tests, it is not clear if this is a general result in hypersonic flow, the result of cold-wall conditions, or simply specific to this experiment. A second observation is that β_u increases with Re_{δ_0}. Since the trend is weak, it is difficult to judge whether this is a real effect or a small systematic experimental error. It does not appear to be the result of viscous effects at the fin leading edge, since the opposite trend with Reynolds number would be expected.

In Fig. 2, the intersection point of each data set with the shock line is different since the Mach angle μ depends on M_∞. Further, in this coordinate system, a characteristic feature of the swept shock flowfield is effectively masked. To bring this feature out, obtain a common origin, and see more directly the influence of M_∞ on the relation between β_u and β_s, the data have been replotted as $\Delta\beta=(\beta_u-\beta_s)$ vs $\beta_s-\mu$ (Fig. 3).

For stronger shocks, $(\beta_s-\mu)> \approx 6$-8 deg, it can be seen that the relationship between $\Delta\beta$ and $(\beta_s-\mu)$ is essentially linear at both Mach 3 and 6. In neither case does the best-fit straight line pass through the origin (this is most apparent at Mach 6). Although the increased data scatter makes it more difficult to

Table 1 Conically symmetric studies analyzed

Mach No.	δ_0, cm	$Re_{\delta_0}\times 10^6$	C_f	α, deg	Refs.
2	0.36	0.24	0.00169	4-8	4
3	0.44	0.29	0.00144	2-20	5, 10
6	0.37	0.12	—	6-16	1
6	0.30	0.30	—	6-16	1

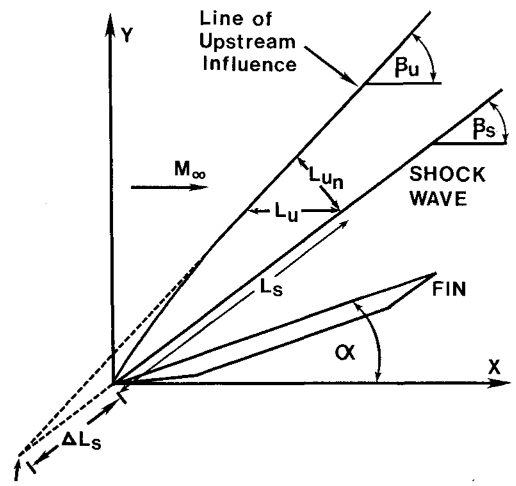

Fig. 1 Model and coordinate system.